

Four Zero Texture Fermion Mass Matrices in $SO(10)$ GUT

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Abstract

We attempt the integration of the phenomenologically successful four zero texture of fermion mass matrices with the renormalizable $SO(10)$ GUT. The resulting scenario is found to be highly predictive. Firstly, we examine the phenomenological implications of a class of the lepton mass matrices with parallel texture structures and obtain interesting constraints on the parameters of the charged lepton and the neutrino mass matrices. We combine these phenomenological constraints with the constraints obtained from $SO(10)$ GUT to reduce the number of the free parameters and to further constrain the allowed ranges of the free parameters. The solar/atmospheric mixing angles obtained in this analysis are in fairly good agreement with the data.

1 Introduction

The origin of the fermion masses and mixings along with the related problem of CP violation constitute a formidable challenge for elementary particle physics. Leaving apart extremely

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small neutrino masses, even the charged fermion mass hierarchy ranges over at least five orders of magnitude. Since the fermion masses are derived from the Yukawa couplings, which are free parameters within the Standard Model (SM), these Yukawa couplings must span several orders of magnitude to accommodate the strongly hierarchical pattern of the fermion masses and mixings. However, the currently available data on the fermion masses and mixings is insufficient for an unambiguous reconstruction of the fermion mass matrices. To make matters worse, radiative corrections can obscure the underlying structures. Thus, the existing data cannot, without some additional assumptions, determine all the elements of the Yukawa coupling matrices for the quarks and the leptons. Some of these assumptions, invoked to restrict the form of the fermion mass matrices include the presence of texture zeros [1], requirement of zero determinant [2] and vanishing minors [3] to name just a few. The main motivation for invoking different mass matrix Ansätze is to relate the fermion masses and mixing angles in a testable manner which reduces the number of free parameters in the Yukawa sector. The recent evidence for non-zero neutrino masses and mixings leads to a further proliferation of free parameters in the Yukawa sector. In the absence of a significant breakthrough in the theoretical understanding of the fermion flavors, the phenomenological approaches are bound to play a crucial role in interpreting new experimental data on the quark and the lepton mixing. These approaches are expected to provide useful hints towards unravelling the dynamics of the fermion mass generation, CP violation and identification of possible underlying symmetries of the fermion flavors from which viable models of the fermion mass generation and flavor mixing could, hopefully, be constructed.

The strong fermion mass hierarchy should be apparent in the fermion mass matrices themselves with the contribution of smaller elements to physical masses and mixing angles expected to be negligibly small. Thus, these elements can, effectively, be neglected and replaced by zeros: the so-called texture zeros. However, the current neutrino oscillation data is consistent only with a limited number of texture schemes [1]. Specifically, the available neutrino oscillation data disallow all the neutrino mass matrices with three or more texture zeros [1] in the flavor basis. The texture zeros at different positions in the neutrino mass matrix, in particular, and the fermion mass matrices, in general, could be the consequence of some underlying symmetry. Such universal textures of the fermion mass matrices can be realized within the framework of Grand Unified Theories (GUTs). Though Grand Unification on its own does not shed any light on the flavor problem, the GUTs provide the optimal framework in which possible solutions to the flavor problem could be embedded. Grand unified models attempt to explain the masses and mixings in both the quark and the lepton sectors simultaneously. The textures for the mass matrices obtained in these models can either be assumed at the very outset or can be derived from the observed mixing matrix in the flavor basis. Alternatively, the textures of the mass matrix can be obtained by embedding some family symmetry within the chosen Grand Unification group. One particularly interesting class of models is that based upon the $SO(10)$ Grand Unification group. There are two

kinds of minimal models in this class: those based upon Higgs dimension **10**, **126**, $\overline{\mathbf{126}}$ and possibly also **120** and/or **210** and those based upon **10**, **16**, $\overline{\mathbf{16}}$ and **45** representations. The former choice, generally, has symmetric and/or antisymmetric texture mass matrices while the latter type generally imply lopsided mass matrices for the down type quarks and the charged leptons. In the present work, we consider the four zero texture [FTZ] Ansatz for the fermion mass matrices within the SO(10) GUT framework. Within this framework, we not only have a relation between the down type quark mass matrices (M_d) and the charged lepton mass matrices (M_l) but also a relation between the up-type quark mass matrices (M_u) and the Dirac neutrino mass matrices (M_D).

Four zero texture Ansatz is especially important since it can successfully describe not only the quark but also the lepton sector including the charged lepton and the neutrino masses. Moreover, these are compatible with specific GUT models [4] and can be obtained from Abelian flavor symmetries [5]. Furthermore, these mass matrices can accommodate the present value of $\text{Sin}2\beta$ [6]. In view of the phenomenological success of the FZT Ansatz, it would be interesting to examine it in the larger context of SO(10) GUT. The FZT Ansatz has been studied earlier [7] in the context of SO(10) GUT which, in general, leads to relations between mass matrices. On the other hand texture zeros imply relations between the elements of the mass matrices. It is, therefore, interesting to examine FZT Ansätze within the framework of SO(10) GUT. In the earlier analysis [7] somehow, the constraints implied by texture zeros of the neutrino mass matrix have not been incorporated into the analysis and only SO(10) constraints on mass matrices have been used. In the present work, we have obtained the FZT constraints on the lepton mass matrices in Sec. 2. These constraints have been incorporated into the analysis of SO(10) GUT with four zero texture structure in Sec. 3. We have also presented the Takagi diagonalization of the general complex symmetric neutrino mass matrix in Sec. 3 without making any simplifying assumption for the phases. Conclusions have been summarized in Sec. 4.

2 Four zero texture lepton mass matrices

All information about the lepton masses and mixings is encoded in a hermitian charged lepton mass matrix M_e and the neutrino mass matrix M_ν which is complex symmetric. Both these matrices have parallel texture structure i.e. zeros at (1,1) and (1,3) places [8]. In the present work, we consider a special case of FZT Ansätze in which the lepton mass matrices have parallel texture structure with zero entries at (1, 1) and (1, 3) places. The charged fermion mass matrices are assumed to be of the form

$$M_f = \begin{pmatrix} 0 & a_f & 0 \\ a_f^* & b_f & c_f \\ 0 & c_f^* & d_f \end{pmatrix} \quad (1)$$

where $f = e, u$ and d for charged lepton, up-type quarks and down-type quarks, respectively. The elements a_f and c_f are complex with $a_f = |a_f|e^{i\phi_f}$, $c_f = |c_f|e^{i\psi_f}$. The elements b_f and d_f are real because of hermiticity of M_f . The unitary matrices V_f diagonalize the mass matrices M_f :

$$M_f = V_f M_f^d V_f^\dagger. \quad (2)$$

Here $M_f^d = (-m_{f1}, m_{f2}, m_{f3})$ and $V_f = P_f O_f$ where $P_f = \text{diag}(e^{i\phi_f}, 1, e^{i\psi_f})$. O_f is the diagonalization matrix for the real symmetric mass matrix $M_f^{(r)} = P_f^\dagger M_f P_f$. Using the invariants, $\text{Tr} M_f^r$, $\text{Tr} M_f^{r^2}$ and $\text{Det} M_f^r$ three elements (a_f, b_f, c_f) of the total four elements can be written in terms of remaining one element d_f and the mass eigenvalues $-m_{f1}$, m_{f2} and m_{f3} of the fermion mass matrix M_f as

$$\begin{aligned} b_f &= -m_{f1} + m_{f2} + m_{f3} - d_f, \\ |a_f| &= \left(\frac{m_{f1} m_{f2} m_{f3}}{d_f} \right)^{\frac{1}{2}}, \\ |c_f| &= \left[-\frac{(d_f + m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f} \right]^{\frac{1}{2}}. \end{aligned} \quad (3)$$

The free parameter d_f should be in the range $m_{f2} < d_f < m_{f3}$ for the elements $|a_f|$ and $|c_f|$ to be real. Using Eqn.(3) the elements of the diagonalization matrix, O_f can be written in terms of the charged fermion masses and the charged fermion mass matrix element d_f as

$$\left. \begin{aligned} O_{f11} &= \sqrt{\frac{m_{f2} m_{f3} (d_f + m_{f1})}{d_f (-m_{f1} - m_{f2}) (-m_{f1} - m_{f3})}} \\ O_{f12} &= \sqrt{\frac{-m_{f1} m_{f3} (d_f - m_{f2})}{d_f (m_{f2} - m_{f3}) (m_{f2} + m_{f1})}} \\ O_{f13} &= \sqrt{\frac{-m_{f1} m_{f2} (d_f - m_{f3})}{d_f (m_{f3} - m_{f2}) (m_{f3} + m_{f1})}} \\ O_{f21} &= -\sqrt{\frac{m_{f1} (d_f + m_{f1})}{(-m_{f1} - m_{f2}) (-m_{f1} - m_{f3})}} \\ O_{f22} &= \sqrt{\frac{m_{f2} (d_f - m_{f2})}{-(m_{f2} - m_{f3}) (m_{f2} + m_{f1})}} \\ O_{f23} &= \sqrt{\frac{m_{f3} (d_f - m_{f3})}{-(m_{f3} - m_{f2}) (m_{f3} + m_{f1})}} \\ O_{f31} &= \sqrt{\frac{-m_{f1} (d_f - m_{f2}) (d_f - m_{f3})}{d_f (-m_{f1} - m_{f2}) (-m_{f1} - m_{f3})}} \\ O_{f32} &= -\sqrt{\frac{m_{f2} (d_f - m_{f3}) (d_f + m_{f1})}{d_f (m_{f2} - m_{f3}) (m_{f2} + m_{f1})}} \\ O_{f33} &= \sqrt{\frac{m_{f3} (d_f + m_{f1}) (d_f - m_{f2})}{d_f (m_{f3} - m_{f2}) (m_{f3} + m_{f1})}} \end{aligned} \right\}. \quad (4)$$

The neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & a_\nu & 0 \\ a_\nu & b_\nu & c_\nu \\ 0 & c_\nu & d_\nu \end{pmatrix} \quad (5)$$

is complex symmetric for which all the four elements are in general complex and is diagonalized by a complex unitary matrix V_ν :

$$M_\nu = V_\nu M_\nu^{diag} V_\nu^T. \quad (6)$$

where $M_\nu^{diag} = \text{diag}(-m_1, m_2, m_3)$. The diagonalization matrices V_e , V_ν , V_u and V_d are not physically observable but the products $U_{CKM} = V_u^\dagger V_d$ [9], and $U_{PMNS} = V_e^\dagger V_\nu$ [10] are the physically observable quark and lepton mixing matrices, respectively. They can be parametrized in terms of the mixing angles and CP violating phases. Considering neutrinos to be the Majorana particles, we parametrize $U_{PMNS} = U.P$ in terms of the three mixing angles (θ_{12} , θ_{23} and θ_{13}) and the three CP violating phases (α , β and δ) as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix} \quad (7)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ for $i, j=1, 2, 3$.

The (1,1) and (1,3) entries of the neutrino mass matrix M_ν are assumed to be zero. Thus, we obtain two complex homogeneous linear equations for m_1 , \tilde{m}_2 and \tilde{m}_3 given by

$$m_1 a^2 + \tilde{m}_2 b^2 + \tilde{m}_3 c^2 = 0, \quad (8)$$

$$m_1 ad + \tilde{m}_2 bg + \tilde{m}_3 ch = 0 \quad (9)$$

where the complex coefficients a , b , c , d , g and h are given by

$$\left. \begin{aligned} a &= O_{e11}U_{e1} + O_{e12}U_{m1} + O_{e13}U_{t1}, \\ b &= O_{e11}U_{e2} + O_{e12}U_{m2} + O_{e13}U_{t2}, \\ c &= O_{e11}U_{e3} + O_{e12}U_{m3} + O_{e13}U_{t3}, \\ d &= O_{e31}U_{e1} + O_{e32}U_{m1} + O_{e33}U_{t1}, \\ g &= O_{e31}U_{e2} + O_{e32}U_{m2} + O_{e33}U_{t2}, \\ h &= O_{e31}U_{e3} + O_{e32}U_{m3} + O_{e33}U_{t3}, \end{aligned} \right\} \quad (10)$$

and $\tilde{m}_2 = m_2 e^{2i\alpha}$, $\tilde{m}_3 = m_3 e^{2i(\beta+\delta)}$. Solving Eqns.(8-9) for the two mass ratios $\left(\frac{m_1}{m_2}\right)$ and $\left(\frac{m_1}{m_3}\right)$, we obtain

$$\begin{aligned} \frac{m_1}{m_2} e^{-2i\alpha} &= \frac{b(cg-bh)}{a(ah-cd)}, \\ \frac{m_1}{m_3} e^{-2i\beta} &= \frac{c(bh-cg)}{a(ag-bd)} e^{2i\delta}. \end{aligned} \quad (11)$$

One can enumerate the number of parameters in Eqn.(11). The nine parameters [three neutrino mixing angles (θ_{12} , θ_{23} , θ_{13}), three neutrino mass eigenvalues (m_1 , m_2 , m_3), two Majorana-type CP violating phases (α , β) and Dirac-type CP violating phase, δ] come from the neutrino sector and the four parameters [three charged lepton masses (m_e , m_μ , m_τ) and free parameter d_e] come from the charged lepton sector, thus, totalling thirteen parameters. The three charged lepton masses are [11]

$$m_e = 0.510998910 \text{ MeV}, m_\mu = 105.658367 \text{ MeV}, m_\tau = 1776.84 \text{ MeV}. \quad (12)$$

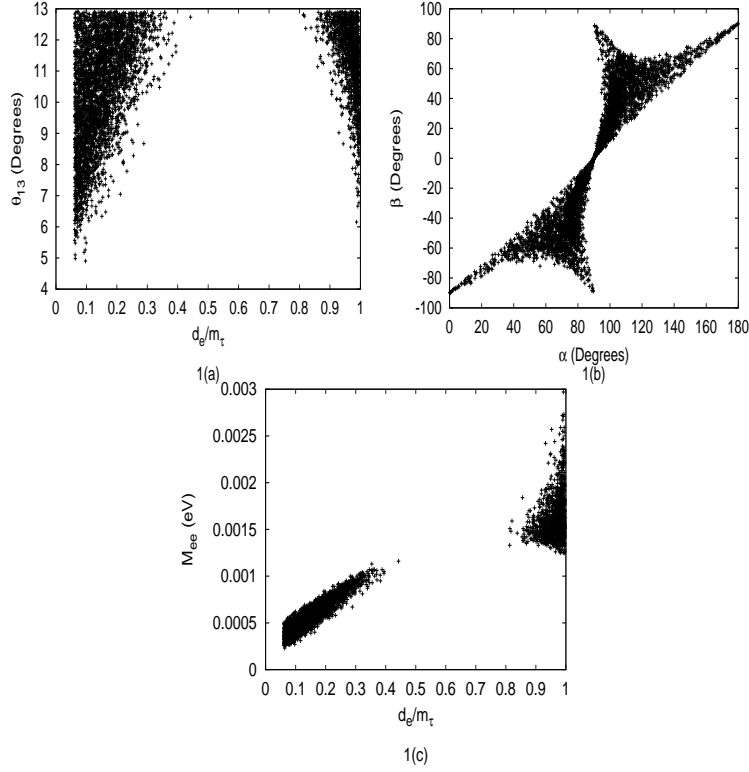


Figure 1: Correlation plots for d_e/m_τ , θ_{13} , M_{ee} and the Majorana phases at 3σ .

The experimental constraints on the neutrino oscillation parameters with their best fit values and 3σ deviations are [12]

$$\begin{aligned}
\Delta m_{12}^2 &= 7.67_{-0.53}^{+0.52} \times 10^{-5} eV^2, \\
\Delta m_{31}^2 &= 2.39_{-0.33}^{+0.42} \times 10^{-3} eV^2, \\
\theta_{12}^o &= 33.95_{-3.1}^{+3.81}, \\
\theta_{23}^o &= 43.05_{-7.93}^{+10.32}.
\end{aligned} \tag{13}$$

In addition, there is an upper bound on the mixing angle θ_{13} from the CHOOZ experiment ($\theta_{12}^o < 12.38$). Recently T2K[13] and MINOS[14] experiments have given hints of a relatively large θ_{13} .

For the simultaneous existence of two texture zeros at (1, 1) and (1, 3) positions in M_ν , the two values of m_1 given by

$$m_1 = \left| \frac{m_1}{m_2} \right| \sqrt{\frac{\Delta m_{12}^2}{1 - \left| \frac{m_1}{m_2} \right|^2}} \tag{14}$$

and

$$m_1 = \left| \frac{m_1}{m_3} \right| \sqrt{\frac{\Delta m_{12}^2 + \Delta m_{23}^2}{1 - \left| \frac{m_1}{m_3} \right|^2}}, \tag{15}$$

calculated from the mass ratios $\left(\frac{m_1}{m_2}, \frac{m_1}{m_3} \right)$, respectively, must be equal to within the errors of oscillation data. This constraint can be used to constrain the unknown parameters δ , d_e and

the third mixing angle θ_{13} which has not been measured experimentally as yet. We, thus, obtain not only the correlations [Fig. 1(a), Fig. 1(b)] amongst the unknown parameters [viz. the neutrino mixing angles, the CP violating phases and d_e] but also constrain the unknown parameter d_e . We obtain two allowed regions for the free parameter d_e . We call them ‘low d_e ’ and ‘high d_e ’ solutions. The other two free parameters (ϕ_e and ψ_e) are not constrained by the presence of texture zeros because they factor out from Eqns.(8) and (9). We have, also, calculated the effective Majorana mass M_{ee} (Fig.1(c)) appearing in the neutrinoless double beta decay for the allowed parameter space

$$M_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|. \quad (16)$$

These constraints on the hitherto unknown parameter d_e obtained from the FZT Ansatz will be used as an input in the next section where we attempt to constrain the SO(10) GUT.

3 Four Zero Texture and SO(10) GUT

In the next step we combine the FZT Ansatz with the constraints obtained from the SO(10) GUT. Such an analysis will require renormalization group (RG) running from weak scale to the GUT scale. However, it is known that the effects of RG running are negligible for mass matrices with normal hierarchy. Having obtained the texture zero constraints on the free parameter d_e , we now further constrain it by imposing the following SO(10) GUT relations

$$\left. \begin{aligned} M_u &= S + \delta'' A + \epsilon S' \equiv S_u + A_u, \\ M_d &= \eta S + \delta' A + S' \equiv S_d + A_d, \\ M_D &= S + \delta''' A - 3\epsilon S' \equiv S_D + A_D, \\ M_e &= \eta S + A - 3S' \equiv S_e + A_e, \\ M_L &= \rho S' \equiv S_L, \\ M_R &= \gamma S' \equiv S_R. \end{aligned} \right\} \quad (17)$$

where S , S' are the symmetric part coming from the **10** and **126** dimensional Higgs representations, respectively, and A is the antisymmetric part coming from the **120** dimensional Higgs representation. Here, η , ϵ , ρ , γ , δ' , δ'' and δ''' represent the relative coefficients of the vacuum expectation values (VEVs). From Eqn.(17), we find that eighteen parameters come from $S_f + A_f$, $f = u, d, e$ and four are the SO(10) coefficients δ' , η , ϵ and δ'' , thus, totalling twenty two free parameters. Eqn.(17) implies the following relations

$$\begin{aligned} 4\eta S_u &= (3 + \eta\epsilon)S_d + (1 - \eta\epsilon)S_e, \\ \delta' A_u &= \delta'' A_d = \delta' \delta'' A_e. \end{aligned} \quad (18)$$

In the component form, the above set of SO(10) constraints (Eqn.(17)) can be written as

$$\left. \begin{aligned} 4\eta a_u \cos(\Delta\phi + \phi_d) &= (3 + \eta\epsilon)a_d \cos\phi_d + (1 - \eta\epsilon)a_e \cos\phi_e, \\ 4\eta c_u \cos(\Delta\psi + \psi_d) &= (3 + \eta\epsilon)c_d \cos\psi_d + (1 - \eta\epsilon)c_e \cos\psi_e, \\ 4\eta b_u &= (3 + \eta\epsilon)b_d + (1 - \eta\epsilon)b_e, \\ 4\eta d_u &= (3 + \eta\epsilon)d_d + (1 - \eta\epsilon)d_e, \\ \delta' a_u \sin(\Delta\phi + \phi_d) &= \delta'' a_d \sin\phi_d = \delta' \delta'' a_e \sin\phi_e, \\ \delta' c_u \sin(\Delta\psi + \psi_d) &= \delta'' c_d \sin\psi_d = \delta' \delta'' c_e \sin\psi_e. \end{aligned} \right\} \quad (19)$$

In the above, we have largely followed the notations and conventions of Ref.[7] which combines the SO(10) constraints in the form of Eqn.(19). We have 8 independent equations on 22 free parameters appearing in the quark and the charged lepton sectors. However, using the experimental results on the quark/charged lepton masses and mixings which includes six quark masses, three CKM mixing angles, one CKM CP violating phase and three charged lepton masses, we are left with only one free parameter to be fixed. These eight independent equations can be used to relate d_e , ϕ_e and ψ_e with the corresponding parameters in the quark sector i.e. d_u , d_d , $\Delta\phi = \phi_u - \phi_d$ and $\Delta\psi = \psi_u - \psi_d$ (we note that only these phase differences are determined from the experimental data). The phases ϕ_e and ψ_e are given by the equations

$$\cos\phi_e \equiv \frac{4\eta a_u \cos(\Delta\phi + \phi_d) - (3 + \kappa)a_d \cos\phi_d}{(1 - \kappa)a_e}, \quad (20)$$

$$\cos\psi_e \equiv \frac{4\eta c_u \cos(\Delta\psi + \psi_d) - (3 + \kappa)c_d \cos\psi_d}{(1 - \kappa)c_e}, \quad (21)$$

where the down type quark phases are

$$\tan\phi_d = \frac{a_u \sin\Delta\phi}{ra_d - a_u \cos\Delta\phi}, \quad (22)$$

$$\tan\psi_d = \frac{c_u \sin\Delta\psi}{rc_d - c_u \cos\Delta\psi}, \quad (23)$$

with $r = \frac{\delta''}{\delta'}$ and

$$\eta = \frac{(m_d + m_s + m_b)d_e - (m_e + m_\mu + m_\tau)d_d}{(m_d + m_s + m_b - m_e - m_\mu - m_\tau)d_u - (m_u + m_c + m_t)(d_d - d_e)}, \quad (24)$$

$$\kappa = \frac{(m_u + m_c + m_t)(3d_d + d_e) - (3(m_d + m_s + m_b) + (m_e + m_\mu + m_\tau))d_u}{(m_d + m_s + m_b - m_e - m_\mu - m_\tau)d_u - (m_u + m_c + m_t)(d_d - d_e)}. \quad (25)$$

The parameter d_e is constrained by the requirement that $\cos\phi_e$ and $\cos\psi_e$ in Eqns.(20-21) are physical and

$$\frac{c_d \sin\psi_d}{a_d \sin\phi_d} = \frac{c_e \sin\psi_e}{a_e \sin\phi_e}. \quad (26)$$

The parameters d_u , d_d , $\Delta\phi$ and $\Delta\psi$ can be constrained by the observed CKM matrix [6].

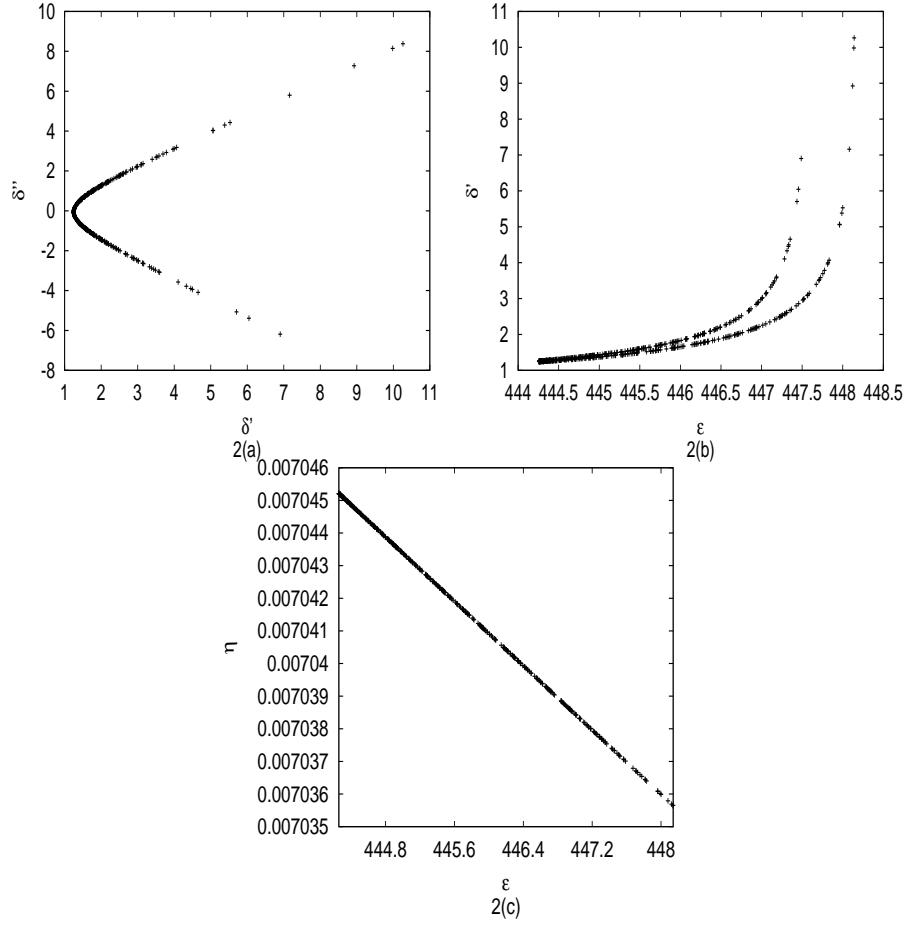


Figure 2: Correlation plots for some of the SO(10) parameters for high d_e .

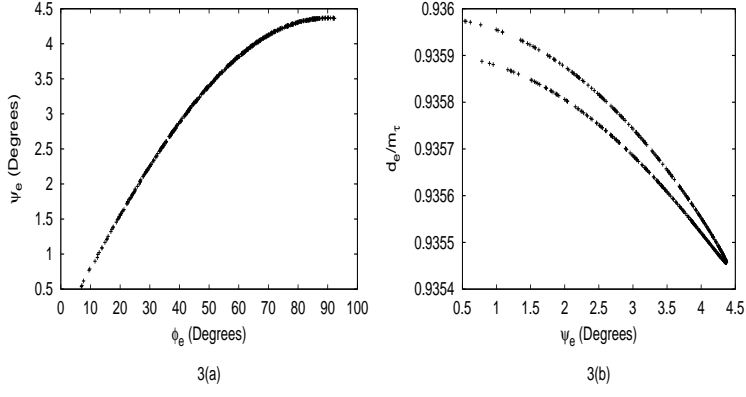


Figure 3: Correlation plots for phases from the charged lepton sector and d_e/m_τ (higher region).

The best fit values realized for these parameters are given by

$$\Delta\phi = \frac{\pi}{2}, \quad (27)$$

$$\Delta\psi = -0.12, \quad (28)$$

$$d_u = 0.95m_t, \quad (29)$$

$$d_d = 0.94m_b. \quad (30)$$

In our numerical analysis, we have used the following best fit values for the quark and the charged lepton masses [7]:

$$\begin{aligned} m_u &= 1.04MeV, m_c = 302MeV, m_t = 129GeV, \\ m_d &= 1.33MeV, m_s = 26.5MeV, m_b = 1.0GeV, \\ m_e &= 0.325MeV, m_\mu = 68.55MeV, m_\tau = 1171.62MeV. \end{aligned} \quad (31)$$

The correlation plots between ϕ_e , ψ_e and d_e/m_τ for the best fit values of d_u , d_d , $\Delta\phi$ and $\Delta\psi$ have been plotted in Fig.(3). In the phenomenological analysis of the FZT Ansatz, presented in the previous section, we obtained two allowed regions for the free parameter d_e compatible with the available data on lepton masses and mixings. Using these phenomenologically allowed ranges for d_e along with the SO(10) GUT relations we constrain the $d_e - r$ plane. After the combined input of zero textures and SO(10) relations, the allowed ranges of d_e are $(0.307346m_\tau < d_e < 0.309016m_\tau)$ for the low d_e and $(0.935427m_\tau < d_e < 0.935991m_\tau)$ for high d_e . So far we have succeeded in fitting the unknown parameters of the charged lepton sector for the best fit values of d_u , d_d , $\Delta\phi$ and $\Delta\psi$.

The neutrino mass matrix from Type-I+II seesaw mechanism [15] is given by

$$M_\nu = M_L - M_D M_R^{-1} M_D^T \quad (32)$$

which introduces three more unknown parameters ρ , γ and δ''' (Eqn.(17)). The lepton mixing matrix is given by

$$U_{PMNS} = V_e^\dagger V_\nu \quad (33)$$

where V_e and V_ν are the diagonalization matrices for the charged leptons and neutrinos. The diagonalization of the charged lepton mass matrix M_e is similar to the quark sector (Eqn.(3)). For Majorana neutrinos, the complex symmetric neutrino mass matrix is diagonalized by a unitary matrix V_ν (Eqn.(6)). Takagi factorization [16] can diagonalize a general complex symmetric neutrino mass matrix without any simplifying assumption for the phases. For a complex symmetric neutrino mass matrix M_ν there exists a unitary matrix V_ν and a real non-negative diagonal matrix M_ν^{diag} such that

$$M_\nu = V_\nu M_\nu^{diag} V_\nu^T \quad (34)$$

where the columns of V_ν are an orthonormal set of eigenvectors for $M_\nu M_\nu^\dagger$. The corresponding diagonal entries of M_ν^{diag} are non-negative square roots of the corresponding eigenvalues of $M_\nu M_\nu^\dagger$. The information about neutrino masses and mixings can be obtained from Eqn.(33)

$$\tan^2 \theta_{12} = \frac{|(U_{PMNS})_{12}|^2}{|(U_{PMNS})_{11}|^2}, \quad (35)$$

$$\sin^2 2\theta_{23} = \frac{4|(U_{PMNS})_{23}|^2|(U_{PMNS})_{33}|^2}{(1 - |(U_{PMNS})_{13}|^2)^2}, \quad (36)$$

and

$$\sin \theta_{13} = |(U_{PMNS})_{13}| \quad (37)$$

In order to obtain consistent values of the lepton mixing angles we have to fine tune the unknown parameters ρ , γ and δ''' . For example, for $d_e = 0.93575m_\tau$, we obtain the following mixing angles

$$\theta_{12} = 33.45^\circ, \quad (38)$$

$$\theta_{23} = 47.92^\circ, \quad (39)$$

$$\theta_{13} = 2.035^\circ. \quad (40)$$

The unknown parameters ρ , γ and δ''' are fine tuned to be

$$\rho = 1.43 \times 10^{-9}, \quad \gamma = 2 \times 10^{16}, \quad \delta''' = 10. \quad (41)$$

We obtain a small value for $R_\nu = \frac{\Delta m_{12}^2}{\Delta m_{23}^2} = 5.37 \times 10^{-4}$, which is well below the experimentally allowed range. However, it must be noted that the mixing angles and mass-squared differences have been calculated for the best fit values of the parameters $\Delta\psi$, $\Delta\phi$, d_u , d_d given in Eqns.(27-30). However, these parameters have wide ranges and one may obtain consistent values of θ_{13} and R_ν when we consider the full ranges of these parameters. This has been illustrated explicitly in the Appendix by picking up some representative values of the parameters from within their allowed ranges to calculate the neutrino mass squared differences and the mixing angles which are in agreement with the currently available data. Our objective in using the best fit values is only meant to illustrate the methodology presented in this work.

4 Conclusions

We discussed the four zero texture quark/lepton mass matrices in the context of SO(10) GUT. The integration of these two scenarios reduces the number of free parameters to one when we incorporate the known data from the quark and the charged lepton sectors. In our analysis, firstly we obtained two regions of solutions for the unknown parameter d_e from the phenomenological analysis for the four zero texture lepton mass matrices. Then we used the SO(10) relations to further constrain the allowed parameter space. We successfully fitted all the unknown parameters from four zero textures and SO(10) GUT within the framework of Type-I+II seesaw mechanism. The solar/atmospheric mixing angles obtained in this analysis are in fairly good agreement with the data. Of course, one must run down the neutrino mass matrix from the GUT scale to the electroweak scale and fit with the known oscillation data. However, it is well known that the effects of RG running are negligible for mass matrices with the normal hierarchy peculiar of the texture structure considered in this work.

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5 Appendix

In this appendix we give the numerical values of different parameters which satisfy the experimental data using the input parameters $\Delta\psi$, $\Delta\phi$, d_u , d_d in there allowed experimental ranges[6]. Table-I lists the numerical values of different parameters. The neutrino mixing angles and mass squared differences for these values of parameters are

$$\theta_{12} = 32.26^\circ, \quad \theta_{23} = 45.74^\circ, \quad \theta_{13} = 12.34^\circ. \quad (42)$$

$$\Delta m_{12}^2 = 7.43 \times 10^{-5} eV^2, \quad \Delta m_{32}^2 = 2.703 \times 10^{-3} eV^2. \quad (43)$$

References

- [1] Paul H. Frampton, Sheldon L. Glashow and Danny Marfatia, *Phys. Lett.* **B 536**, 79 (2002), hep-ph/0201008; Zhi-zhong Xing, *Phys. Lett.* **B 530**, 159 (2002), hep-ph/0201151; Bipin R. Desai, D. P. Roy and Alexander R. Vaucher, *Mod. Phys. Lett* **A 18**, 1355 (2003), hep-ph/0209035; S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, *Nucl. Phys.* **B 784**, 103-117 (2007), hep-ph/0611313; S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, *Phys. Rev.* **D 76**, 013002 (2007), hep-ph/0612102.

Parameter	Numerical value
d_u	$0.951934 m_t$
d_d	$0.932640 m_b$
$\Delta\phi$	$\pi/2$
$\Delta\psi$	-0.044058
d_e	$0.890355 m_\tau$
r	0.408963
ρ	3×10^{-10}
γ	2.238477×10^{18}
δ'''	29414

Table 1: Numerical values of parameters

- [2] G. C. Branco, R. Gonzalez, F. R. Joaquim and T. Yanagida, *Phys. Lett.* **B 562**, 265 (2003), hep-ph/0212341; Bhag C. Chauhan, Joao Pulido and Marco Picariello, *Phys. Rev.* **D 73**, 053003 (2006), hep-ph/0602084.
- [3] E. I. Lashin and N. Chamoun, *Phys. Rev.* **D 78**, 073002 (2008), arXiv:0708.2423 [hep-ph]; E. I. Lashin, N. Chamoun, *Phys. Rev.* **D 80**, 093004 (2009), arXiv:0909.2669 [hep-ph]; S. Dev, Surender Verma, Shivani Gupta and R. R. Gautam, *Phys. Rev.* **D 81**, 053010 (2010), arXiv:1003.1006 [hep-ph]; S. Dev, Shivani Gupta and R. R. Gautam, *Mod. Phys. Lett.* **A 26**, 501-514, arXiv:1011.5587 [hep-ph]; S. Dev, Shivani Gupta, Radha Raman Gautam and Lal Singh, *Phys. Lett.* **B 706**, 168 (2011), arXiv:1111.1300 [hep-ph].
- [4] H. Fritzsch and Z. Z. Xing, *Prog. Part. Nucl. Phys.* **45**, 1 (2000), hep-ph/9912358 and references therein.
- [5] W. Grimus, *Proc. Sci.*, **HEP 186** (2005), hep-ph/0511078.
- [6] K. Matsuda and H. Nishiura, *Phys. Rev.* **D 74**, 033014 (2006), hep-ph/0606142 and references therein.
- [7] Takeshi Fukuyama, Koichi Matsuda and Hiroyuki Nishiura, *Int. J. Mod. Phys.* **A 22**, 5325-5343 (2007), hep-ph/0702284; K. Matsuda, H. Nishiura and T. Fukuyama, *Phys. Rev.* **D 61**, 053001 (2000), hep-ph/9906433.
- [8] S. Dev, Sanjeev Kumar, Surender Verma and Shivani Gupta, *Mod. Phys. Lett.* **A 24**, 28(2009), arXiv:0810.3083 [hep-ph]
- [9] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

- [10] B. Pontecorvo, *Zh. Eksp. Teor. Fiz. (JTEP)* **33**, 549 (1957); *ibid.* **34**, 247 (1958); *ibid.* **53**, 1717 (1967); Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962).
- [11] C. Amsler *et al.* [Particle Data Group], *Phys. Lett. B* **667**, 1 (2008).
- [12] Proceedings of NO-VE 2008, IV International workshop on "Neutrino Oscillations in Venice" (Venice, Italy, April 15-18, 2008), edited by M. Baldo Ceolin (University of Padova publication, Papergraf Editions, Padova, Italy, 2008), pages 21-28, hep-ph/0809.2936
- [13] K. Abe et al. [T2K collaboration], *Phys. Rev. Lett.* **107**, 041801 (2011), arXiv:1106.2822 [hep-ex].
- [14] P. Adamson et al. [MINOS collaboration], *Phys. Rev. Lett.* **107**, 181802 (2011), arXiv:1108.0015 [hep-ex].
- [15] W. Konetschny and W. Kummer, *Phys. Lett. B* **70**, 433 (1977); T. P. Cheng and L. F. Li, *Phys. Rev. D* **22**, 2860 (1980); J. Schechter and J. W. F. Valle, *Phys. Rev. D* **22**, 2227 (1980); G. Lazarides Q. Shafi and C. Wetterich, *Nucl. Phys. B* **181**, 287 (1981); R. N. Mohapatra and G. Senjanovic, *Phys. Rev. D* **23**, 165 (1981).
- [16] T. Takagi, *Japan J. Math.* **1**, 83 (1923); R. A. Horn and C. R. Johnson, *Matrix Analysis* (Cambridge University Press, Cambridge, England, 1990).